CHARACTERISTICS OF STOCHASTIC VOLATILITY FOR LATIN AMERICA'S *PAR BONDS*[1]

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ABSTRACT

This paper uses a quasi-maximum likelihood procedure to estimate the nonstationary stochastic volatility for the par bonds of four Latin American countries (Brazil, Argentina, Mexico and Venezuela). The aim is to investigate the possible presence of co-movements in volatility across these countries. The estimation period goes from August 1994 until September 1999, including therefore the Asian and Russian crises. The estimated volatility for the univariate model does not show any slope and is highly persistent. The multivariate model gives a good fit to the data and shows that there is common movement.

Key Words: Stochastic Volatility, Co-Movements; Maximum Likelihood
JEL classification: C32

Introduction

The two oil crises in the 70's, and the combination of low economic growth of industrial countries, the world's highest interest rates and the decrease in commodity prices in the world market resulted in a higher debt for the developing countries which were already deeply indebted.

The debt crisis in the early 80's started after the Mexican government suspended the payment of its commercial debts to world banks. As a result, the international financial market refused to grant new credit to developing nations, aggravating even more the economic conditions in these countries.

The 1985 Baker Plan was the first attempt to restructure the debts of developing countries. The plan called for commercial banks and multilateral organizations to extend new loans, and for the debtor countries to enact reforms reducing economic instability, accelerating growth and cutting down on pending debts. However, the plan failed to achieve its desired effect mainly due to its short term horizon for the payment of debt securities (maximum 2 years) and also because of the recession those countries had to cope with at that time.

A new attempt was made in 1989 with the Brady Plan, which aimed at restructuring external debt services in a way that not only the principal but also interest rates could be reduced. Thus, there

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would be a greater financial flexibility, allowing market liquidity of such debts through their securitization, and also allowing economic agents to disseminate risk in the secondary market. As before, debtor countries were obliged to make economic adjustments according to a program devised by the IMF. This program, known as SAP (*Structural Adjustment Program*), set up targets for inflation, GDP growth, privatizations and decentralization of governmental services. The changes imposed by the program were intended to promote investments, internal reserves of capital and capital repatriation.

The main difference between Brady and Baker Plans is that the former tried to reduce external debt while the latter was only concerned with the extension of new credit.

As Brady Plan negotiations were dealt with on a case-by-case basis between debtor countries and creditors, including three possible options for restructuring debts[4], different securities with distinctive characteristics were created.

In February 1990, Mexico became the first country to renegotiate its external debt using Brady Plan patterns, that is, converting US$ 48 billion of its external debts into two loan options within negotiable guarantees: the *Discount Bonds* and the *Par Bonds*. Successively, twelve other countries in Asia, Latin America, Africa and East Europe adopted this strategy[5]. Brazil, Mexico and Argentina accounted for two thirds of the Brady Bonds issued[6].

Most of these bonds are expressed in U.S. dollars, 70% of them have a maturity over 10 years, and are categorized into fixed floating income instruments. Brady Bonds turned out to be quite attractive as their principal is *collateralized*[7] in U.S. Treasury bonds, with fixed or floating interest rates, offering attractive rates of return to investors, allowing for portfolio diversification, and easy negotiation in the secondary market. The most marketable and common are the *Par Bonds* and the *Discount Bonds*, for having an average long life and amortization prospects.

The prices of Brady Bonds are influenced by the economic conditions in the country of issue as well as in the collateral. Changes in the growth path of debtor countries cause market uncertainty affecting sovereignty risk, due to the increase in *default*[8] probabilities. Changes in the market of U.S.

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[4] The options were to trade loans for securities, use securities with face value maturity and fixed interest rate or discount bonds and floating rate or a new loan in which payment was restructured.

[5] These countries were: Brazil, Argentina, Venezuela, Bulgaria, Costa Rica, Dominican Republic, Philippines, Uruguay, Morocco, Nigeria and Ecuador.

[6] Among Brady bonds we find: Par Bonds, Discount Bonds (DB), Debt Conversion Bonds (DCB), New Money Bonds (NMB), Front-Load Interest Reduction Bonds (FLIRB´S), Capitalization Bonds (C-Bonds) and Interest Due Unpaid (IDU).

[7] *Collateral* is an asset that serves as a guarantee to a creditor until the loan is paid off. If the debtor becomes insolvent, the creditor has the legal right to capture the pledged assets and sell them in order to pay off the loan.

[8] Izvorski (1998) calculates the implicit *default* probability for the prices of Brady Bonds of seven developing countries.

Treasury bonds (*collateral*), international interest rate oscillations, and changes in the relative return of bonds, as was the case of Mexico's currency devaluation in 1994 and Brazil's in 1999, wind up affecting the price of Bradies.

These bonds have become a thermometer to international investors, measuring the performance of emergent nations, being sensitive to investment risk variations in these emergent countries[9]. Therefore, there is a naturally higher volatility associated with the assets of these countries, especially in terms of external debt securities.

In recent years there have been several structural adjustments targeted at stabilizing emergent economies. Nevertheless, as this process involved increased costs, many governments had difficulty preventing exchange rates and internal interest rates from moving up.

As far as international investors are concerned, emergent countries usually present similar problems with regard to their economic fundamentals. It is commonly believed that when a certain country is faced with an economic crisis, there is a high probability of dissemination of such crisis into other regions (contagious effect). Thus, we may conclude that there is a common perception as far as risk amidst emerging economies is concerned.

The recent succession of financial crises, as those which assailed Mexico in 1994; Asia in 1997; Russia in 1998 and Brazil in 1999, stirred debate on this risk relationship between emerging economies, either for their contagious effect or for the existence of a correlation[10].

When those crises took place, governments of other countries rushed to declare that the economic status of their countries was totally different, which should be read by investors as non-existence of correlation between their assets, and that the contagious effect was then eliminated. As a matter of fact, that was not what could be observed, especially in Latin America.

If assets are not really correlated and there is no contagious effect, that is, if foreign investors believe these markets are relatively independent, the volatility of Bradies with equivalent characteristics between these countries, as for instance, *Par Bonds*, is not expected to present common movement over time.

The present paper tries to show how the volatility relationship expresses itself for the *Par Bonds* of four Latin American countries (Mexico, Brazil, Argentina and Venezuela) using a multivariate stochastic volatility method.

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[9] For example, many analysts measure "Brazil's risk" by means of *Spread over Treasury*, that is, how many basis points above a U.S. Treasury bond with similar maturity a *C-Bond* is negotiated in the market

Par Bonds are used for having long-term maturity and liquidity, and for being issued by the four countries in question, allowing us to carry out an analysis of risk correlation regarding a common asset. It is worth mentioning that the payment of Mexico's *Par Bonds* coupons is bound to the future price of oil and to the profits obtained from oil exportation. *Par Bonds* are negotiated at a 100% face value, with a lower market price *coupon.* The principal is usually collateralized by a zero-coupon U.S. Treasury bond, whose price will eventually become a floor price for the value of Brady bonds.[11]

Table 1 – Par Bonds Characteristics

Country	Date of Issue	Principal in US$ Billion	Currency	Semiannual Coupon	Maturity
Argentina	04/1993	14.9	US$, DM	Flexible	04/2023
Mexico	03/1990	22.6	US$, Y, DM	6.25%	12/2019
Brazil	04/1994	8.4	US$, DM, L	Flexible	04/2024
Venezuela	12/1990	6.7	US$, DM, FFr	6.75%	03/2020

The objective of this paper is, therefore, to determine the stochastic components and the common characteristics involved in *Par Bonds* volatility. The paper is divided into 4 sections. The first section discusses the characteristics of univariate stochastic volatility models whereas the second section deals with multivariate models. After the presentation of the econometric models, the third section describes the characteristics of the data and their use, along with the estimation results. The last section provides the conclusions of our study.

1. Univariate Stochastic Volatility

An univariate structural time series model, as shown by Harvey (1997), may be formulated as in equation (1.1),

$$y_t = \mu_t + \gamma_t + \psi_t + \varepsilon_t \qquad t = 1,....,T \qquad (1.1)$$

[10] If volatility in the financial market of a given country is high, an investment fund could sell its securities of that country or any other correlated to it and purchase assets from countries that are not undergoing an economic crisis. This situation generates a contagious effect to other countries and regions around the world.

where ε_t has a zero mean and variance σ_ε^2, and the unobservable components are given by level (μ_t), seasonality (γ_t) and cyclic component (ψ_t). Consider that an asset return *(R_t)* is given by

$$R_t = \sigma_t \varepsilon_t \qquad t = 1,\dots T \qquad \varepsilon_t \sim NID(0,1) \qquad (1.2)$$

R_t, volatility (h_t) may be determined as an unobservable component that presents a certain characteristic of time evolution. Thus, by having returns squared and the log extracted, we obtain;

$$R_t^2 = \sigma_t^2 \varepsilon_t^2$$

$$\log R_t^2 = h_t + \log \varepsilon_t^2 \quad \text{where } h_t = \log \sigma_t^2 \qquad (1.3)$$

Observe that $\log \varepsilon_t^2$ presents $\log(\chi_{(1)}^2)$ distribution and that $E(\log \varepsilon_t^2) = -1.27$ and $E[(\log \varepsilon_t^2)^2] = \pi^2/2$.

Now consider an innovation process ξ_t given by $\xi_t = \log \varepsilon_t^2 - E(\log \varepsilon_t^2)$, in a way that $E(\xi_t) = 0$ and $Var(\xi_t) = \pi^2/2$. Since $E(\log \varepsilon_t^2) = -1.27$, then $\log \varepsilon_t^2 = \xi_t - 1.27$. Therefore[12], (1.3) may be altered in order to obtain

$$\log R_t^2 = h_t + \xi_t - 1.27 \qquad (1.4)$$

With (h_t) as an unobservable component, its time evolution may follow, for example, an autoregressive process of order 1, like

$$h_t = \phi h_{t-1} + \eta_t \qquad (1.5)$$

Thus, equations (1.4) and (1.5) represent the stochastic volatility model in state space form where (h_t) stands for unobservable component or, in other words, stochastic variance. If $\phi=1$, then h_t is a random walk and the best linear predictor for h_t current values is an EWMA (*Exponentially*

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[11] A *coupon* is an interest rate on a representative debt security , in which the issuing part agrees to pay the holder within the established term, expressed by the security face value percentage. A security without interest coupon is called *zero-coupon*.

Weighted Moving Average)[13] of previous $\log R_t^2$ values. Observe that there is a relationship to IGARCH[14] deterministic model here.

We use Kalman filter to estimate hyperparameters $\phi, \sigma_\varepsilon^2$ and σ_η^2 by a quasi-maximum likelihood method, as the model does not have a Gaussian distribution. Therefore, we have

$$LogL_Q(R/\theta) = -\frac{n}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{n}\log F_t - \frac{1}{2}\sum_{t=1}^{n}\frac{v_t^2}{F_t} \quad (1.6)$$

where $R=(R_1,.....,R_n)$ are the returns, v_t is the one step-ahead prediction error for the best linear estimation of $\log R_t^2$, while F_t is the mean squared error and θ is the vector of unknown parameters. In this case, as $\log \varepsilon_t^2$ does not have a normal distribution, the quasi-likelihood estimator is a suboptimal estimator.

2. Multivariate Stochastic Volatility

Multivariate structural time series models are interesting as they can reveal associations among the series being used. This relationship may be observed through the error correlation of unobservable components in multivariate models. In addition to this, multivariate models are so flexible that they allow imposing certain restrictions so as to obtain a common trend, a common cycle or a common slope[15], or homogeneous error covariance matrices (this model is known as homogeneous system). These covariance matrices correspond to the variance parameters in the univariate model. On the main diagonal, we find the variances of the corresponding disturbances, above the diagonal are the correlations and below the diagonal the covariances.

This way, the multivariate structural model with time-correlated components is similar to SUTSE *(Seemingly Unrelated Time Series Equations)*. However, as there may be a correlation between the series errors, in the SUTSE model each series may be modeled as in the univariate

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[12] For detailed information on the univariate stochastic volatility model, see Ruiz (1994), Herência (1997) or Morais and Portugal (1999).

[13] See Harvey (1996)

[14] In the structural model, $\phi=1$ is equivalent to $\alpha_1 + \beta_1 = 1$ in case of GARCH (1,1), where $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}$. See Engle and Bollerslev (1986).

[15] The presence of common factors means that covariance matrices of relevant errors are smaller than the full rank. For further explanations on how to decompose these models, see Harvey and Koopman (1997), or Harvey (1996).

model.[16] If the error covariance matrices are proportional, which means that the series have the same dynamic properties, [17] then the SUTSE model is homogeneous.

The generalization of univariate stochastic volatility for the multivariate case is relatively simple. Take *N* series of returns, in a way that $\boldsymbol{R_t}$ is a vector *Nx1*. Therefore, there will be a vector $\boldsymbol{\varepsilon_t}$ *Nx1* with irregular components, producing a covariance matrix $\boldsymbol{\Sigma_\varepsilon}$ for $\boldsymbol{\varepsilon_t}$ components vector. Consider that this vector $(\boldsymbol{R_t})$ of returns obeys the following condition

$$R_{it} = \sigma_{it}\varepsilon_{it} \qquad t = 1,....,T \qquad i = 1,......,N \qquad (2.1)$$

in which R_{it} is the observed return series *i* in time *t*, and ε_{it} is the irregular component *i* in time *t* of a vector $\boldsymbol{\varepsilon_t}$ *Nx1* with irregular components, with mean zero and covariance matrix Σ_ε, in which the elements on the diagonal are 1 and the off-diagonal elements are represented by ρ_{it}. Observe that equation 2.1 is equation 1.2 multivariate case.

Following the univariate model and generalizing the AR(1) process of the component variance for the *N* series, $h_{it} = \phi_i h_{it-1} + \eta_{it}$, where h_{it} stands for the stochastic variance of series *i* in time *t*, the following formulation in state space form is obtained for the multivariate case

$$\log R_{it}^2 = -1{,}27\lambda + h_t + \xi_t \qquad (2.2)$$
$$h_t = \phi h_{t-1} + \eta_t$$

where $\log R_{it}^2$ and $\boldsymbol{\xi_t}$ are vectors *Nx1* with $\xi_{it} = \log\varepsilon_{it}^2 + 1.27$ and λ is a vector *Nx1* of numbers 1. Here, the set of equations (2.2) is similar to the set (1.4) and (1.5) and, likewise, it does not have a Gaussian distribution. Therefore, a quasi-maximum likelihood estimator may be used in order to obtain the hyperparameters for the model. The quantity of such parameters depends on whether restrictions are or not imposed. [18]

In this paper, for each return series there is an equation like

$$R_{it} = \alpha_0 + \sum_{p=1}^{T}\alpha_p R_{it-p} + \sum_{q=0}^{Z}\beta_q D_q + o_{it} \qquad i = 1,....,N \qquad (2.3)$$

[16] See Harvey (1996)
[17] The same autocorrelation function for the stationary model array.
[18] ξ_t and η_t are unrelated in the univariate model as well as in the multivariate model. See Harvey (1994).

where R_{it} stands for the return, D_q are dummy variables (intervention) and o_{it} are the residuals. Once the residual vector $o_t = (o_{1t},......,o_{NT})'$ is obtained and supposing these residuals are given by

$$o_{it} = \sigma_{it}\varepsilon_{it} \qquad t = 1,....T \qquad i = 1,......, N \qquad (2.4)$$

the multivariate stochastic variance model may be then formulated in a similar way to the set of equations (2.2)[19].

It is necessary to eliminate o_{it} values that equal zero (as in 2.4) since these values do not allow log operator application. One alternative is to subtract the mean value from o_{it}, as in Harvey Ruiz e Shephard (1994). Another alternative is to use the equation

$$\log o_{it}^2 \cong \log(o_{it}^2 + cS_{o_i}^2) - cS_{o_i}^2 \big/ (o_{it}^2 + cS_{o_i}^2) \qquad (2.5)$$

which complies with a transformation based upon Taylor series, $S_{o_i}^2$ is the sample variance of o_i and c is a small value parameter[20]. One advantage of using Kalman filter is that it allows obtaining either filtered or smoothed stochastic volatility estimations.

As smoothed volatility is obtained by considering all available information, and as the main objective of our study is not to make an accurate prediction but to detect the existence of common characteristics within the series, considering the smoothed stochastic volatility allows us to obtain inference gains. By using $\log o_{it}^2$, the variance of equation (2.3) residuals may be obtained by the application of (2.6),

$$Vol_{it,p} = \exp(N_{it} + 1{,}27 + h_{it,p}) \qquad (2.6)$$

where p= *smoothened or filtered*, N_{it} is the level of series i and h_{it} is the volatility estimation.

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[19] The estimation process for the multivariate case is, as observed, a generalization of the univariate case. Detailed information on the univariate case is found in Morais and Portugal (1999) or in Herência (1997). There is an explanation on multivariate stochastic volatility models in Koopman (1995), Harvey (1996) and Harvey, Ruiz and Shephard (1994).

[20] The statistical package used was Stamp 5.0, where this transformation is proposed, with a value of 0.02 for c in the program *default*.

3. Data Characteristics

This study is based on the prices of *Par Bonds* for four Latin American countries, Mexico, Brazil, Argentina and Venezuela, between August 9th, 1994 and September 15th, 1999[21], amounting to 1261 data.

As graph 3.1 is going to show, the prices of these bonds have a similar behavior throughout this period. The impact caused by economic crises that assailed Mexico and Asia, the moratorium in Russia and, on top of that, the change of Brazilian exchange rate regime, is quite evident during this period. These facts altogether lead us to believe that, a priori, there must be a correlation among the prices of *Par Bonds* in these countries. Such correlation may mean a unique perception of risk by foreign investors in relation to external debt securities in Latin American countries.

The returns of these bonds are calculated using formula 3.1. The behavior of *Par Bonds* returns for the four countries analyzed are shown in Graph 3.2 and the statistics of the four series are found in Table 3.1.

$$R_{it} = \log P_{it} - \log P_{it-1} \qquad (3.1)$$

here *i=1,2,3,4* where, R_{1t}, R_{2t}, R_{3t} and R_{4t} stand for returns in time *t* presented respectively by Argentina, Mexico, Brazil and Venezuela.

As could be observed, all series are non-normal, presenting an average of return with values that fall close to zero and a standard deviation that is relatively high for the average found. The highest standard deviation was presented by the returns of Argentina's *Par Bonds* whereas the lowest one was presented by Mexico. The Q-Ljung-Box test carried out for returns and squared returns suggests the investigation of existing time dependence in second moments. This characteristic may be modeled from the deterministic formulations as, for instance, GARCH multivariate models. However, here, we are going to use stochastic models since they allow determining the series common characteristics[22].

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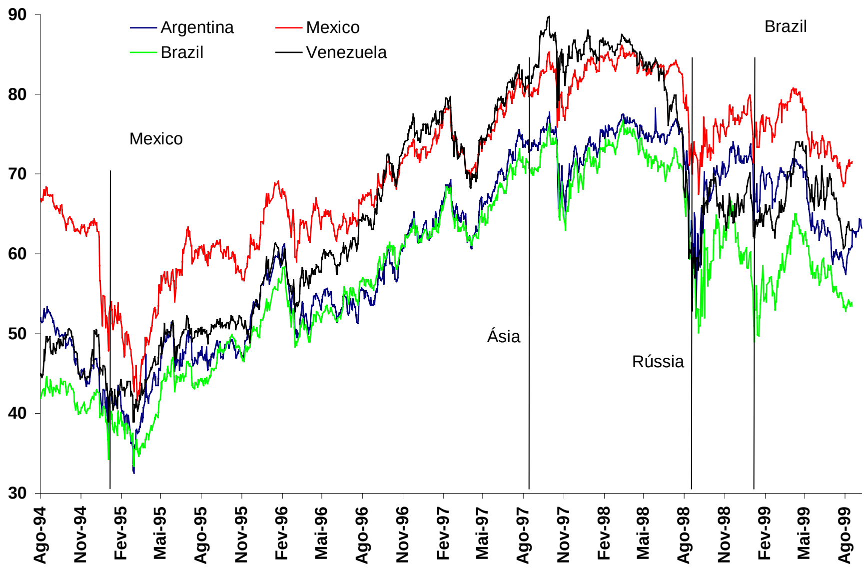
[21] The estimation period ended before the Ecuadorian government declared that it would not pay interests on its Bradies.

[22] Racine and Ackert (1998), use M-Garch model to analyze the behavior of three share indexes in the U.S. market and their associated future prices.

Gráfico 3.1. - Par Bonds Purchase Price



Source: Gazeta Mercantil

Graph 3.2 Par Bonds Returns

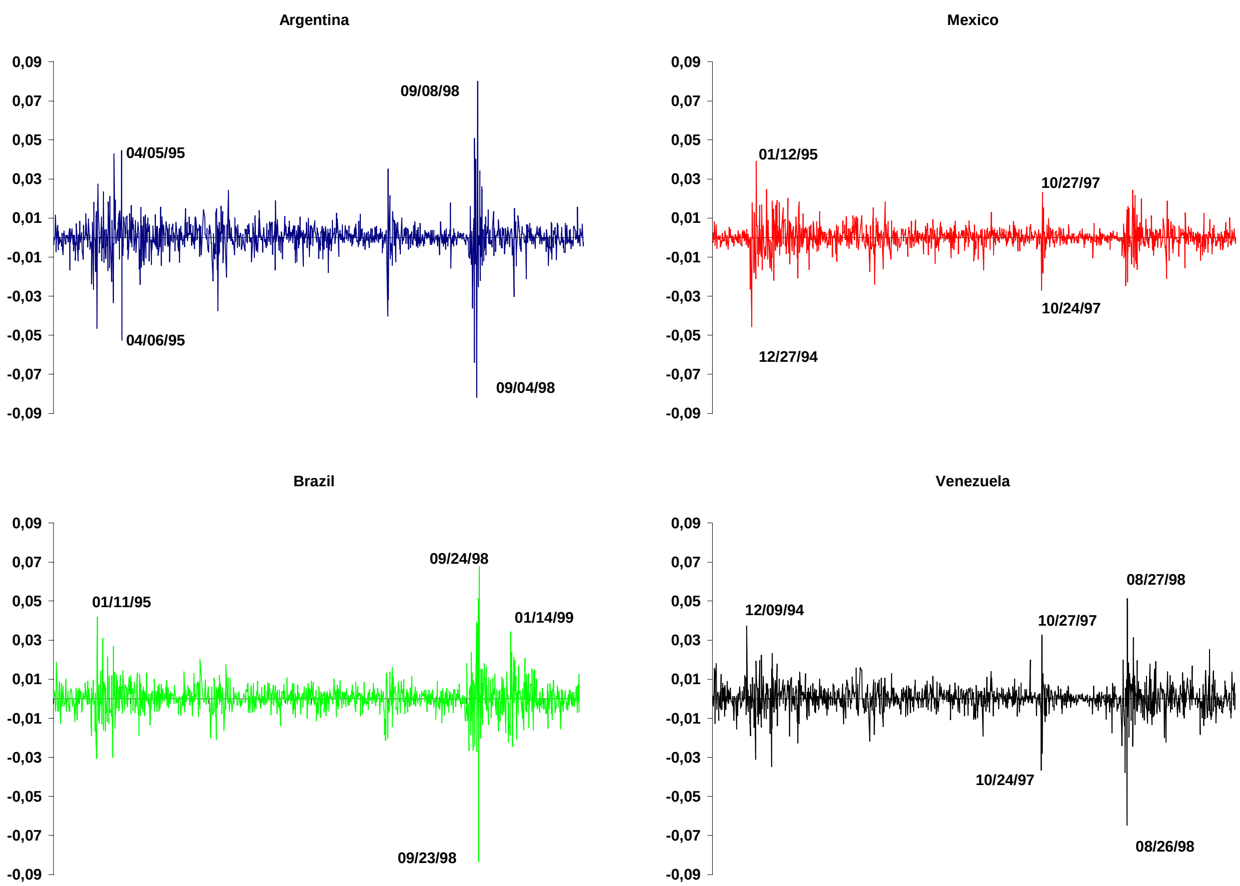


Table 3.1 – Return Series Statistics

	Mexico	Brazil	Argentina	Venezuela
Mean Value	3.138E-05	0.0001	6.75E-05	0.00012
Standard Deviation	0.005738	0.00765	0.00825	0.006816
Normality Test	910.96**	2857.9**	3189.9**	1581**
Maximum Return	0.038918	0.067543	0.079877	0.051267
Minimum Return	-0.045438	-0.083337	-0.08161	-0.064741

Note: (**) Hypothesis H_0 with significant normality at 1%.

In spite of not being included in the formulation, the equation of the mean value that will be used for each series is important for the production of unrelated residuals, which will be used to calculate the stochastic volatility according to equations (2.3) through (2.5). Even though financial series equations in AR(1) format are quite common, the tests revealed the presence of different lag autocorrelations. In addition, in periods of crisis in emergent countries, the returns oscillate quite regularly, entailing extreme values. In order to eliminate this effect, we use outlier dummy variables. Equations (3.2 – 3.5) present the best relationships for the return series of the countries analyzed[23]. The standard deviations are found between parentheses.

$$R_{1t} = -\underset{(0,028)}{0.058} R_{1t-2} - \underset{(0,028)}{0.05888} R_{1t-3} + \underset{(0,028)}{0.08632} R_{1t-4} - \underset{(0,002)}{0.03981} D1 + \underset{(0,003)}{0.0462} D2 + o_{1t} \quad (3.2)$$

$$R_{2t} = -\underset{(0,028)}{0.0934} R_{2t-2} - \underset{0,028)}{0.0577} R_{2t-4} - \underset{(0,003)}{0.0371} D1 + \underset{(0,003)}{0.0298} D2 + o_{2t} \quad (3.3)$$

$$R_{3t} = -\underset{(0,028)}{0.06855} R_{3t-2} - \underset{(0,028)}{0.06359} R_{3t-3} + \underset{(0,007)}{0.057} D1 - \underset{(0,007)}{0.08121} D2 + o_{3t} \quad (3.4)$$

$$R_{4t} = -\underset{(0,028)}{0.06689} R_{4t-1} - \underset{(0,028)}{0.0538} R_{4t-4} - \underset{(0,004)}{0.06362} D1 + \underset{(0,004)}{0.04321} D2 + o_{4t} \quad (3.5)$$

All coefficients are significant. Table 3.2 shows Ljung-Box test for the residuals in the 4 series. None of the residuals are autocorrelated.

Table 3.2 –Ljung-Box Test for Residuals

Q(lag)/Series	Mexico	Brazil	Argentina	Venezuela
Q(1)	0.1359**	0.0262**	0.9787**	0.007255*
Q(2)	0.1366*	0.0433*	0.993**	0.1231*
Q(3)	0.5068**	0.0816*	1.1197**	0.1547*
Q(4)	0.5089*	1.2486**	1.12**	0.1585*
Q(5)	2.45**	1.6709**	1.31**	1.6544**

Note: Hypothesis H_0 with significant non-autocorrelation at 1% (**) and at 5% (*).

[23]In all series, *D1* represents an outlier dummy variable for the negative returns, with different days for each series, and *D2* represents the outlier dummy variables for positive returns.

After finding the vector of residuals, $o_t = (o_{1t}, o_{2t}, o_{3t}, o_{4t})$, a transformation is made using equation (2.5), which eliminates residuals with zero value, to obtain a vector h_t, where $h_{it} = \log o_{it}^2$. We then estimate the stochastic volatility. To begin with, univariate models are used, in order to check the volatility slope for each return series[24]. Table 3.3 presents the results obtained.

Except for Venezuela, the autoregressive coefficient of order 1 for all other series were negative and all log-likelihood maximum values were very close. Mexico's series values were the highest (-761,66). As observed, the slope component, σ_{slope}^2, is zero for the four series, which shows that the slope is deterministic instead of stochastic. Table 3.4[25] shows the values for the model without slope.

Table 3.3 - Univariate SV Models – Slope Test

Components	Mexico	Brazil	Argentina	Venezuela
$\sigma_{irregular}^2$	1.6911	2.7574	0.0000	3.1257
σ_{level}^2	0.1558	0.0265	0.0344	0.00181
σ_{slope}^2	0.0000	0.0000	0.0000	0.0000
σ_{η}^2	0.349	0.2491	3.071	0.0632
AR(1) Coefficient	-0.593	-0.3515	-0.041	0.944
Likelihood Log	-761.68	-763.54	-778.99	-792.02

Table 3.4 - Univariate SV Models

Components	Mexico	Brazil	Argentina	Venezuela
$\sigma_{irregular}^2$	2.867	2.434	0.6841	3.129
σ_{level}^2	0.0232	0.0256	0.03302	0.000978
σ_{η}^2	0.1182	0.592	2.3911	0.0621
AR(1) Coefficient	-0.597	-0.1644	-0.0496	0.948
Likelihood Log	-755.75	-757.74	-773.26	-784.77

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[24] The model in state space form used as slope test looks like ;

$$\begin{aligned} o_{it} &= N_{it} + h_{it} + \xi_{it} \\ N_{it} &= N_{it-1} + \beta_{it-1} + \delta_{it} \\ \beta_{it} &= \beta_{it-1} + \upsilon_{it} \\ h_{it} &= \phi h_{it-1} + \eta_{it} \end{aligned}$$

[25] Table 3.4 shows the estimation results of the model ;

$$\begin{aligned} o_{it} &= N_{it} + h_{it} + \xi_{it} \\ N_{it} &= N_{it-1} + \delta_{it} \\ h_{it} &= \phi h_{it-1} + \eta_{it} \end{aligned}$$

Observe now that, subtracting σ^2_{slopes}, the irregular component for Argentina goes from 0.00 to 0.6841, and AR(1) coefficients of Mexico, Argentina, Venezuela and Brazil still have values that are very close to those obtained from the previous model. However, as return series do not seem to have a stochastic level[26], a fixed level model could be estimated. This formulation will be referred to as stationary model since there is imposition of restriction $|\phi| < 1$[27] in equation (1.5) of the stochastic variance. The results are shown in Table 3.5.

Table 3.5 - Univariate SV Models - Stationary

Components	Mexico	Brazil	Argentina	Venezuela
$\sigma^2_{irregular}$	3.0381	3.019	3.074	3.147
σ^2_{η}	0.0315	0.0389	0.0451	0.0532
AR(1) Coefficient	0.9803	0.973	0.970	0.963
Likelihood Log	-752.87	-753.00	-767.91	-785.10

Observe that the AR(1) coefficients of all series are quite close to 1, which reveals high persistence in *Par Bonds* volatility. Mexico's coefficient is the highest (0.98) and also presents the highest log-likelihood value. This way, by imposing the restriction that $\phi = 1$, in other words, assuming that the model is nonstationary, in which $h_{it} = h_{it-1} + \eta_{it}$, the stochastic variance changes into a random walk. The results for this estimation are shown in Table 3.6.

Table 3.6 - Univariate SV Models – Nonstationary

Components	Mexico	Brazil	Argentina	Venezuela
$\sigma^2_{irregular}$	3.065	3.0647	3.106	3.2263
σ^2_{η}	0.0219	0.0232	0.03037	0.02513
AR(1) Coefficient	1	1	1	1
Likelihood Log	-756.86	-758.27	-774.037	-791.29

The components found in the nonstationary model, either for their irregular part or volatility are close to those of the stationary model whereas the log-likelihood value for the nonstationary model estimations are slightly lower than those of the stationary estimation (Table 3.5). The next

[26] Stationarity is defined by the unit root test.

[27] Having a fixed level, the model follows the pattern presented in footnote 24, only considering that $N_{it} = N_{it-1}$.Thus, $\sigma^2_{nível}$ =0, that is, the level is deterministic.

step is the estimation of the multivariate models, which are formulated following a persistent movement in volatility, where h_t is a multivariate random walk.

The unit root tests (Phillips Perron and ADF) applied to the four series indicate that all of them are stationary, which was already to be expected due to the use of equation 3.1. However, the reliability of these tests when the reduced form of the set of equations 2.2 is used, is questionable mainly if there is high persistence, that is, AR(1) coefficient is close to 1, as is the case of the results obtained here.

With a coefficient value quite close to 1, the unit root tests reject the null hypothesis of the unit root, often because it is very hard to distinguish the model from a white noise[28]. Therefore, following Harvey, Ruiz and Shepard (1994), we have not used Johansen (1988) procedure. Instead, we applied local level multivariate models as described in equation 3.6, from which it is possible to conclude the number of common trends by analyzing the main component of the estimated matrix $\hat{\Sigma}_\eta$.

$$\log o_{it}^2 = -1{,}27\lambda + h_t + \xi_t \qquad (3.6)$$
$$h_t = h_{t-1} + \eta_t$$

In this case, two models are estimated. In model 1, we have a full rank $\hat{\Sigma}_\eta$ matrix, in which h_t is *NxK* with *K=N*. In model 2, a restriction is made on this matrix in order to obtain common factors of the series variances by using equation 3.7,

$$\log o_{it}^2 = -1{,}27\lambda + \theta h_t + \bar{h} + \xi_t \qquad (3.7)$$
$$h_t = h_{t-1} + \eta_t$$

where θ is an *NxK* coefficient matrix, with $K<N$, h_t and η_t are vectors *Nx1*, $\bar{h}$ is a vector *Nx1* with *K* first elements being zero, and *N-K* being unrestricted[29].

Therefore, by estimating the local level multivariate unrestricted model (equation 3.6), we obtain the covariance matrices for the residuals. The covariances matrices are shown below:

[28] See Pantula (1991) and Schwert (1989)

[29] *N is* the number of dependent variables, which in the present case is 4, while *K* determines the number of common factors.

$$\hat{\Sigma}_{\xi} = \begin{bmatrix} 3.072 & 0.1101 & 0.3924 & 0.08638 \\ 0.3331 & 2.977 & 0.0449 & 0.27652 \\ 1.189 & 0.1340 & 2.988 & 0.04785 \\ 0.2704 & 0.8521 & 0.1477 & 3.190 \end{bmatrix} \quad \hat{\Sigma}_{\eta} = \begin{bmatrix} 0.0377 & 0.9998 & 0.9739 & 0.9675 \\ 0.0395 & 0.0415 & 0.9739 & 0.9675 \\ 0.0359 & 0.0376 & 0.0360 & 0.9841 \\ 0.0345 & 0.0362 & 0.0343 & 0.0337 \end{bmatrix}$$

On the main diagonal are the variances, above the diagonal are the correlations and below the diagonal are the covariances.[30]. Argentina and Mexico present a high correlation as to the volatility level *(* $\rho_{\eta_{ii}} = 0.9998$ *)* while the other correlations are a bit lower and quite close. In general, η_t correlations are higher than ε_t correlations (in the comparison between the triangle above the main diagonal). This high correlation that exists in $\hat{\Sigma}_{\eta}$ is an indicative that the *Par Bonds* volatilities have a common time relationship. The log-likelihood value for the multivariate model (-2787.02) is much higher than the univariate model logs sum (-3080.45), presenting a strong convergence after 28 iterations, having considered 20 hyperparameters.

The analysis of the principal components in $\hat{\Sigma}_{\eta}$ estimation allows predicting the number of possible common trends *(k)*, where the presence of common factors implies the existence of cointegration. The results of $\hat{\Sigma}_{\eta}$ main component analysis are shown in Table 3.7. In this analysis, the volatility covariance matrix is decomposed as $\hat{\Sigma}_{\eta} = \Theta D \Theta'$, where Θ is the matrix of eigenvectors and *D* is a diagonal matrix of eigenvalues.[31]

Table 3.7 – Principal Component Analysis

Eigenvalues	0.1942	0.0000	0.1899	0.1837
Variance %	34.2	0	33.4	32.3

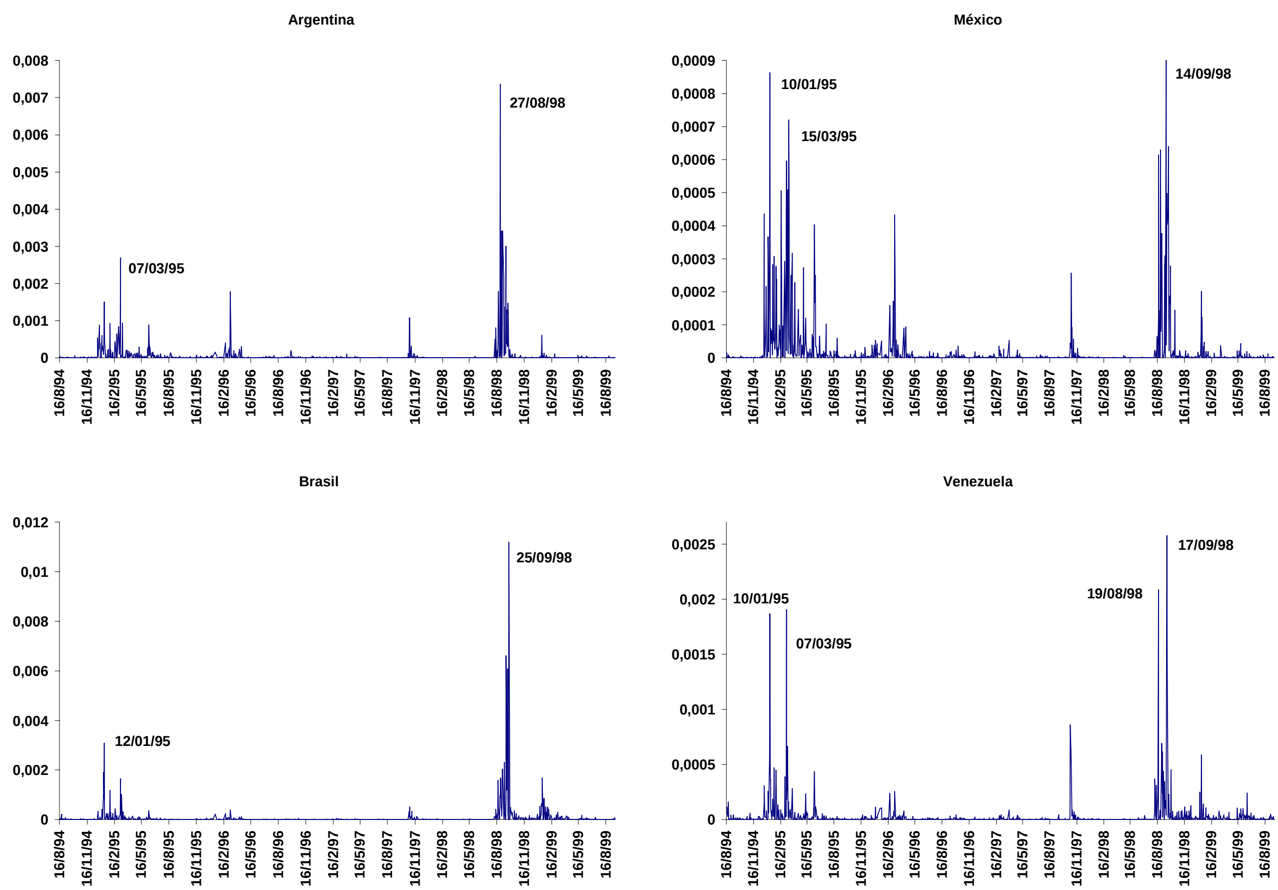
The 3 eigenvalues different from zero are close, explaining approximate percentages. Therefore, we can assume that *K=3*, and then estimate model 2, which produces the following covariance matrices,

$$\hat{\Sigma}_{\xi} = \begin{bmatrix} 3.072 & 0.1104 & 0.3931 & 0.0838 \\ 0.3341 & 2.980 & 0.0434 & 0.2725 \\ 1.192 & 0.1297 & 2.995 & 0.0427 \\ 0.2637 & 0.8442 & 0.1326 & 3.220 \end{bmatrix} \quad \hat{\Sigma}_{\eta} = \begin{bmatrix} 0.0379 & 0.9998 & 0.9719 & 0.9861 \\ 0.0397 & 0.0416 & 0.9719 & 0.9861 \\ 0.0357 & 0.0374 & 0.0357 & 0.9974 \\ 0.0350 & 0.0366 & 0.0343 & 0.0332 \end{bmatrix}$$

[30] The columns and rows of matrices obey the sequence of the series: Argentina, Mexico, Brazil and Venezuela.
[31] The diagonal matrix of eigenvalues *D* obtained is [0.947 0 0.1889]

Here, *N=4* and *K=3* in a way that $\log o_{it}^{2}$ series are co-integrated following the models of Engle and Granger (1987). In other words, there is *N-K=1* a linear combination of $\log o_{it}^{2}$ which represents white noise. The correlations between irregular components are low, as in model 1. Again, there is a great correlation between Argentina's and Mexico's stochastic volatility series component while all other correlations are greater than those obtained in model 1. The log-likelihood maximum value for this restricted model is slightly higher than the previous one (-2788.07), but, even so, quite higher than the log sums of the univariate nonstationary models. Due to the restriction imposed, 19 hyperparameters were estimated. Convergence was very strong in 29 iterations.

Graph 3.2 - Stochastic Volatility for the model with N=4 e K=3



Using equation 3.7, the relationship between *Par Bonds* volatilities of the four countries can be written as

$$\begin{aligned}\log o_{1t}^2 &= -1.27 + \hat{h}_{1t} \\ \log o_{2t}^2 &= -1.27 + 1.048\hat{h}_{1t} + \hat{h}_{2t} \\ \log o_{3t}^2 &= -1.27 + 0.943\hat{h}_{1t} + 0.01163\hat{h}_{2t} + \hat{h}_{3t} \\ \log o_{4t}^2 &= -1.27 + 0.9236\hat{h}_{1t} + 0.01139\hat{h}_{2t} + 0.0106\hat{h}_{3t} - 0.781\end{aligned} \qquad (3.8)$$

where $\bar{h} = 0.781$ for $\log o_{4t}^2$, and $\bar{h} = 0$ for the others, as the K first elements in $\bar{h}$ are zero and the remaining *N-K* are unrestricted. As the data are expressed in logarithmic functions, we may apply 2.6 in order to obtain the smoothed stochastic volatility [32].

4. Conclusion

The multivariate stochastic volatility model is a generalization of the univariate case, in which it is possible to question the existence of common trends among the variances. In the present paper, we used the series of *Par Bonds* purchase prices of four Latin American countries, Mexico, Brazil, Argentina and Venezuela, between August 9th/1994 and September 15th/1999.

By initially estimating univariate models, it is possible to conclude that the slope is deterministic, i.e., a fixed component. It is also possible to conclude that there is a high persistence in volatility due to the values obtained for ϕ, where the quasi-maximum likelihood approach is used. The multivariate structural formulation applied here is an alternative of that proposed by Johansen (1988) in the presence of high persistence for the series under consideration, allowing us to determine the number of common factors that are present in the model. Thus, it is considered that volatility follows a random walk process in multivariate models, with $\phi=1$.

The covariance matrices are initially formulated without restrictions, that is, with full rank. These matrices are used for analyzing the main components and drawing a conclusion on the number of common factors. It was observed that there is a high correlation between Argentina's and Mexico's *Par Bonds* volatilities, and that, by and large, all the other correlations for the other series

[32] A model in which N=4 and K=2 was also estimated, assuming the existence of 2 common factors. The value for the likelihood of this model (-2789) is close to that obtained from 3 common factors, and the correlations are quite close. The relationship between volatilities is given by

$$\begin{aligned}\log o_{1t}^2 &= -1.27 + \hat{h}_{1t} \\ \log o_{2t}^2 &= -1.27 + 1{,}041\hat{h}_{1t} + \hat{h}_{2t} \\ \log o_{3t}^2 &= -1.27 + 0.945\hat{h}_{1t} + 0.01158\hat{h}_{2t} - 0.5534 \\ \log o_{4t}^2 &= -1.27 + 0.9224\hat{h}_{1t} + 0.0113\hat{h}_{2t} - 1.022\end{aligned}$$

are high and way above the correlations found in irregular components. The following estimation considers that *K=3*, where *K*= number of common factors.

As the results obtained for the correlations in which *K=4* and *3* are quite close, it is possible to conclude that there is a common movement between volatilities. This way, investors are expected to have the same perception of risk when analyzing the four countries as a whole, and trying to decide whether they should invest in one of them. Thus, it is possible that an economic crisis in Latin America affecting the external debt securities of a certain country has an effect on the volatility of other assets with similar characteristics. If, on the one hand, a greater oscillation of asset prices may bring chances of profit from applications, on the other hand, it does jeopardize negotiations, bringing about a risk that could be spread among the assets of the country or continent under consideration.

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