Present Valuing Credit Default Swaps: a Practitioner View

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Summary

In this article we describe a *credit default swap* (CDS) and show how default probabilities (DP) appear in the evaluation of the instrument. We already use the optimization algorithm described in Martin et al [1] to get the DP from the CDS market. Finally we show how to use the implied DP's to give the present value of a CDS contract a bank has in its portfolio.

1) Introduction

In the last few years the market for credit derivatives has experienced a huge growth. These instruments are being actively used not only for hedging purposes but also as a way to improve return on capital.

A bank might use credit derivatives to manage its portfolio of risk. Moreover with a credit derivative a bank can sell credit exposure and still keep a good relationship with an important client.

One of the most actively traded credit derivatives is a credit default swap. A CDS provides insurance in the event of default (called a *credit event*) of a particular company (called the *reference entity*).

To understand how a CDS functions let's consider an investor who has bought a 10-year bond from a company XYZ. In order to protect himself/herself of a default event on the bond the investor might want to buy a 10-year CDS from a counterpart on the specific bond he/she has bought. Under the CDS contract the investor has to pay a certain amount of money every quarter (which depends on the CDS rate on the date the contract has been bought) until the end of the contract or until a credit event happens.

In case a credit event happens before the maturity of the bond the investor will deliver the bond in return for par (we will be assuming a simple plain vanilla CDS). In case no credit event happens before the maturity of the bond the investor will have lost the quarterly paid fee.

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To evaluate the fee payment to be made by the CDS holder one has to first estimate the probability of default of the reference entity. This is in fact the most important step in evaluating any credit derivative security. In this work we show in a simplified way how this is done in practice and we refer the interested reader to the appropriate literature. For simplicity we will be assuming no default risk from the protection seller.

An important feature of the CDS market is that in many ways it has become a lot more liquid than the market for the reference entities. Another important characteristic can be seen by the following practical case: when accessing the credit risk involved in lending to a company it is a common practice to use the rating given by credit rating agencies. A rating is then supposed to be related to the probability of default of a company in honouring its debts, and it is then used to determine credit risk and cost of capital.

Contrary to what has been written above we give the following example: on Apr 11th at 5:39 PM the bid offer for a 3-year CDS on British Telecom (rating A2/A) was 113/124 bp, while the bid offer for a 3-year CDS on France Telecom (rating A3/A-) was 104/115. As a CDS is a traded security we assume that it is a far more reliable source of the instantaneous market view of the default probability of a company than the one offered by credit rating agencies.

In this article we show how default probabilities enter in the determination of a CDS rate. We follow the approach described in Martin et al [1]. For the evaluation of default probabilities from the bond market we refer to the work of Hull and White [4]. For a general overview of credit derivative instruments we refer the interested reader to Tavakoly [5]. For the latest model for credit derivatives pricing we refer to Schonbucher [6].

In Section 2 we show what is a CDS and how to relate the term structure of default probabilities with the credit spread swap rate. How to use the implied probabilities of default to price a credit spread swap is shown in section 3. In section 4 we give a numerical example and in section 5 we give the conclusions.

2) Credit Default Swaps[2]

Suppose that two counter parties B (buyer) and S (seller) enter in a plain vanilla CDS. In general in a T-year plain vanilla CDS the buyer B accepts to pay a fixed amount K_T to the seller S of the CDS in exchange for protection in case of default on a bond issued by a certain company

[2] In what follows, for simplicity, we will not take into account of any accrued interest in the time of default. Moreover we will assume that the CDS seller has probability zero of defaulting on its CDS obligation. For adjusting the equations to include probability of default from the CDS seller we refer to Hull and White [7].

(reference entity). In what follows we assume that in case of a default event the protection seller will pay the non-recoverable value of the reference asset to the CDS buyer at the time of default.

Consider then a contract whose notional is given by N, and assume that upon default the value of the contract is α N, where α is called the *recovery rate* (supposed known a priori). I.e. at the time of default the bond holder would loose (1 – α) N if it did not buy any protection. Or in another way: (1 – α) N is the payoff of the CDS contract at the time of default. Assuming α known, the premium K_T to be paid periodically by the protection buyer will depend on the market view of the probability of default by the reference entity.

Under the terms of the contract the CDS buyer periodically pays a fee (e.g. every quarter) until the end of the contract or until a certain credit event happens, when the company will have defaulted. Therefore the present value (PV) of the cash flows paid by the CDS buyer is the discounted value of each cash flow weighted by the probability that the entity will not have defaulted up to the date of payment of the cash flow.

Thus consider that in the CDS the premiums are paid at a set of dates (normally every quarter) that we will represent by $\Gamma = t_1, t_2, \ldots t_n$. Therefore in a T maturity CDS starting today (t_0) and having quarterly payments, the payment dates are: $t_1 = t_0 + 3$ months, $t_2 = t_0 + 2 * 3$ months, …, and $\tau_n = T$. Assume that $Q(t_i)$ is the probability as seen today that the company will default at time t_i. In this way $1 - Q(t_i)$ is the probability of the reference entity not defaulting by time t_i. Then the expectation of the PV of the cash flows to be paid by the CDS buyer is given by:

$$PV_{Fees} = \sum_{i=1}^{n} D(t_i) \cdot K \cdot N \cdot (1 - Q(t_i)) \qquad (1)$$

where $D(t_i)$ is the t_i risk free discount factor, K is the CDS rate and N is the contract notional.

Let's now look at the same contract from the side of the protection seller. The present value of the payment the CDS seller may have to make in case of default is given by the discounted value of the recovered fraction weighted by the probability of default occurring at each payment date. The default weight used in each date is the following one: in order that default occurs at time t_{i+1} it is necessary that the company has not defaulted up to time t_i and it has defaulted at time t_{i+1}. The weight in question is the product of the probability of being alive up to time t_i and the conditional probability of default between time t_i and t_{i+1}. In this way the PV of the recovered part in case of default is given by:

$$PV_{\mathrm{Recov}ered} = (1-\alpha) \cdot \sum_{i=0}^{n} 0.5 \cdot (D(t_i) + D(t_{i+1})) \cdot N \cdot (1 - Q(t_i)) \cdot q(t_i, t_{i+1}) \qquad (2)$$

where $q(t_i,t_{i+1})$ is the conditional (forward) probability of defaulting between t_i and t_{i+1} given that default did not occur up to time t_i. As an approximation we have used as discount factor the average of the discount factors of the interval in which default may occur.

In order to avoid arbitrage, at the beginning of the contract, eq. 1 and eq. 2 should have the same value. The CDS rate is then given by:

$$K \cong \frac{(1-\alpha)\cdot \sum_{i=0}^{N} 0.5\cdot(D(t_i)+D(t_{i+1}))\cdot(1-Q(t_i))\cdot q(t_i,t_{i+1})}{\sum_{i=1}^{N}(1-Q(t_i))\cdot D(t_i)} \qquad (3)$$

The number of different maturities for which one has CDS rates available in the market will depend on the reference entity. With those rates and a bootstrap method or a minimization algorithm (see Martin et al [1] or Garcia et al [2]) the view of the market of the term structure of forward default probability along time[3] can be implied. In this work we have used the second approach due to its advantages over the bootstrap method (see reference [1] for more detail).

We now open a parenthesis to interpret the factors in $(1\text{-}Q(t_i)) * D(t_i)$ which appears in the equations above. In the literature the term $1 - Q(T)$ (the probability of being alive (PBA) at time T) is related to the so called *hazard rate of default* at time T ($\lambda(T)$) by the following equation:

$$1-Q(T)=e^{-\lambda(T)\cdot T} \qquad (4)$$

If the continuous short rate at time T is given by *r(T),* then the product of the PBA and the discount factor at time T is given by:

$$(1-Q(T))\cdot D(T)=e^{-(r(T)+\lambda(T))\cdot T} \qquad (5)$$

And if we define the *adjusted for default short rate* $R(T) = r(T) + \lambda(T)$, we see that the product in eq. 5 is just the adjusted for default (risky) discount factor (we refer e.g. to Duffie and Singleton [8] for more detail).

[3] There is a recursive relation between Q and q given by:

$$Q(t_{i+1}) = Q(t_i) + (1 - Q(t_i)) \cdot q(t_i,t_{i+1}), \qquad Q(t_0) = 0$$

In the next section we show how to obtain the PV of a CDS which has already begun.

3) Pricing an Existing CDS

Consider a CDS contract with a time to maturity T, which pays (quarterly) X_T rate (yearly) on a notional N. The contract has begun at time t_{CDS} and we are now at time t_0 ($t_0 > t_{CDS}$). The question is what is the present value (PV) of this contract?

The first step is to determine the term structure of default probabilities observed in the market at the day of the evaluation (t_0). Assume that we have access to a set of 1-year, 2-year, and n-year CDS rates $K_1, K_2, \ldots, K_n$ respectively, where $T < t_0 + n$ years. One then applies equation (3) to imply the term structure of default probabilities.

In what follows we will consider the PV of the contract for the seller. Consider then that in the actual contract the next payments dates are at times $\tau_1, \tau_2 \ldots \tau_M$, where $\tau_M = T$. The PV of the fee the seller might still receive is given by (see eq. 1):

$$PV_{Fee} = \sum_{i=1}^{M} D(\tau_i) \cdot K \ \cdot N \cdot (1 - Q(\tau_i)) \qquad (6)$$

where $Q(\tau_i)$'s have been evaluated by interpolation in the set of $Q(t_i)$'s obtained in the calibration phase.

Under the terms of the contract the seller might have to pay a lump of money in case the reference entity defaults. The PV of this payment is given by (the underscore rec refers to possible recovered payments by the protection buyer):

$$PV_{\text{Rec}} = (1-\alpha) \cdot \sum_{i=0}^{M-1} 0.5 \cdot (D(\tau_i) + D(\tau_{i+1})) \cdot N \cdot (1 - Q(\tau_i)) \cdot q(\tau_i, \tau_{i+1}) \qquad (7)$$

From eq.'s 6 and 7 the PV of the CDS for the seller is given by:

$$PV_{CDS} = PV_{Fee} - PV_{\text{Rec}} \qquad (8)$$

In the next section we give a numerical example of the procedure.

4) CDS contract : Numerical Example

Consider a fictitious CDS contract offering protection to default of a company XYZ[4]. The contract start date was 1 Jan 2001 and the expiry date is 1 Oct 2003. The contract rate is 0.50% per annum and the payment dates are 1st Apr, 1st Jul, 1st Oct and 1st Jan of each year (if the contract begins at 1st Jan 2001 the first payment will be made at 1st Apr 2001). The contract notional is 5,000,000 Eur. We will be assuming recovery rate of 30%. The question is what is the present value (PV) of this contract?

Suppose we are at 27th Apr 2001 and the yield curve is given in table 1.

Dates	Discount Factor
27-Apr-2001	1.00000000
30-Apr-2001	0.999599744
1-May-2001	0.999466481
8-May-2001	0.998530636
15-May-2001	0.997600414
1-Jun-2001	0.995356626
2-Jul-2001	0.991271967
1-Aug-2001	0.987417248
1-Nov-2001	0.976166472
1-Feb-2002	0.965490858
1-May-2002	0.955022139
2-May-2002	0.954894785
2-May-2003	0.912588153
3-May-2004	0.86961669
2-May-2005	0.827172688

Table 1 Risk Free Discount Factor at Evaluation Date (27th Apr 2001)

[4] The contract and all the data here presented are fictitious.

The credit spreads used in the calibration algorithm is given in table 2 and the calibration has been done applying the algorithm described in Martin et al [1] (an analysis of the minimization algorithm used is described in Garcia et al [2]).

The default probabilities at the payment dates of the actual contract is shown in table 3 obtained after interpolating from the default probabilities implied from the CDS rates from table 2.

Expiry Date	CDS Spread
27-Apr-2002	80 bp
27-Apr-2003	100 bp
27-Apr-2004	120 bp
27-Apr-2005	140 bp
27-Apr-2006	160 bp

Table 2 Market CDS Spreads

Dates	Probability
1-Jul-2001	0.0018431
1-Oct-2001	0.0044886
1-Jan-2002	0.0072815
1-Apr-2002	0.0103392
1-Jul-2002	0.0138397
1-Oct-2002	0.0177713
1-Jan-2003	0.0220810
1-Apr-2003	0.0267550
1-Jul-2003	0.0317992
1-Oct-2003	0.0371462

Table 3 Cumulative Probability at Payment dates of the CDS contract

The present value of the fees is obtained from eq. 6 with the data in tables 1, 2 and 3. The PV_{Fee} in question is 57,804.77 EUR. The present value of the expected recovered amount in case of default is calculated from eq. 7. The PV_{Rec} value is 122,079.11 EUR. The present value of the

CDS (PV_{CDS}) is –64,274.33 EUR. As already mentioned the negative signal assumes we are looking at the PV for the protection seller.

5) Conclusions

In this article we have described what is a CDS contract, how to imply default probabilities from the CDS rates and how to use the calculated default probabilities to price a running CDS contract through a numerical example.

We have also given an example of two companies from the same industrial sector (telecom) in which the one with better rating had higher expected probabilities of default than the one with a lower rating. This fact might indicate that the CDS market is the one in which the view of the market about default probabilities is actively traded. How this sort of information is to be integrated in a Credit Risk management system or in a reduced form model for credit options is still an actively area of research.

The next step in this work is to integrate the default probabilities just determined in a Duffie Singleton model to evaluate credit options [3].

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