

# A Pricing Model for Sovereign Bond

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## *Abstract*

*Similar to the modeling used to evaluate corporate bonds, where it is a put option on corporate assets, we model sovereign bonds. Instead of company's assets as underlying assets, we use foreign exchange reserves. The results show a fundamental pricing model for sovereign bond and an optimum relation between the debt size, term, mix between floating and fixed interest payments, and size of reserves. The model is applied to a Brazilian Brady Bond.*

The lack of domestic savings to finance new projects or higher domestic consumption in the short (or even long) term has compelled developing countries to borrow abroad. At the beginning of the century this was done mostly via bonds and this international transfer of finance was interrupted during the Great Depression with a generalized default in the developing countries. After the war these countries returned to international financial markets, but this time via bank loans. A new default crisis emerged during the eighties when Mexico defaulted in 1982. This crisis spread through most debtors, culminating in renegotiation when debt was exchanged for bonds. Most of these deals were known as the *Brady plan*<sup>1</sup>. Some time after the negotiation these same countries regained access to new and voluntary money via bond issuance.

The major difference between borrowing by bond and loan, as Eichengreen and Portes (1986) point out, is that under bond finance a default is necessary to start a renegotiation, while in the case of loan finance the default can be avoided with the help of the banks until some rescheduling is reached. In fact, this happens because in loan finance the banks have their balance sheet directly involved, so they are interested in a peaceful solution<sup>2</sup>. Many times it is only a problem of liquidity shortage and banks will certainly manage to provide money and avoid default. Another problem that appears when trying to avoid a default in the bond finance case is the difficulty to put together a large number of lenders and avoid free-riders. Loan finance involves less information problem, because banks in general have more resources to collect information on countries abroad. Nevertheless, nowadays this problem has diminished due to the work of the International Monetary Fund (IMF) and a large number of investment banks that provide information to their investors. The problem of short-term finance has also diminished, because the IMF and many other governmental<sup>3</sup> institutions have acted as last-resort lenders during these times.

Sovereign debt shows a basic difference in comparison to regular corporate debt, which is the impossibility of seizing any asset located inside the issuer country. Therefore, most of the country's net worth (except what is located outside) is worthless for the lender. There is no way to guarantee that it will pay its debt, even if it has enough money. Eaton *et al* (1986) show that the problem of enforcement, moral hazard and adverse selection are worse in sovereign than in corporate debt. There are two basic issues in sovereign debt: ability and willingness to pay.

The ability to pay, or solvency, is a cash-flow analysis of the country's external account. It can have different degrees of sophistication and may include or not a dynamics for interest rate, external supply of funds, current account, etc. It also depends on the government effort to collect revenues and change them for hard currency to service their debt. An extreme version of this case is given by Simonsen (1985), who defines the solvent borrower as the one whose trade account present value is larger than the current debt value.

The willingness to pay is in fact a choice between the loss that comes from a default and the money saved on debt payment. This loss is proportional to the penalties<sup>4</sup> that borrowers believe will be imposed by the lenders, and the latter's willingness to lend again in the future. Bulow and Rogoff (1989b) show that penalties rather than reputation defines the ceiling for debt. They also show in Bulow and Rogoff (1989a) that this ceiling should decrease with the possibility of debt rescheduling. In fact, as Eichengreen and Portes (1986) show, shortly after a default, countries re-enter the voluntary market. Knowing this, lenders shall limit the maximum amount available for sovereign borrowers to the present value of the possible penalties these countries can suffer in case of default.

Although willingness to pay poses the strongest limit on the debt size, Grossman and Van Huyck (1988) show that default has been associated with adverse economics shocks, generating what they call excusable default. So in practice, ability to pay has been the greatest factor to affect sovereign debt prices. Ability to pay is easier to quantify than willingness, and most pricing models are based on ability constraint.

The evaluation of sovereign debt has been made in three different ways, the first a reduced form, where the debt price or Spread Over Treasury (SOT)<sup>5</sup> is explained by a set

of external and internal economic variables without an theoretical connection. Edwards (1984), Purcell and Orlanki(1988) and Bohmer and Megginson (1990) are examples of this pricing technique. A second approach is to find a process for the default<sup>6</sup> and based on this process, find the risk neutral price of the debt. The problem with this approach is that it is obtained with some existing debt, therefore is useful only for arbitrage among different types of debt. Another approach, a more structural one, presents a relationship between fundamental variables and debt price (or SOT).

Corporate debt pricing literature<sup>7</sup> uses the same approaches above, whereas Duffie and Singleton (1996) model the corporate default process as in the second case above. The structural approach, is obtained via an option model, where the debt is worth the expected value of the minimum between asset and bond face values. This kind of model was first developed by Merton(1974), who assumed a stochastic behavior for the corporate asset in a non-stochastic interest rate economy. More recently, Amin and Jarrow(1992) generalized this solution and priced option on risky assets in a stochastic interest rate economy. An application for corporate bonds is in Das (1995), and Longstaff and Schwartz (1995).

The application of option technology to sovereign debt has already been done. Cohen (1993) adopts a close formulation to evaluate sovereign loan. He treats the loan price as the minimum between loan face value and the present value of the potential penalties<sup>8</sup> that can be imposed on the sovereign debtor. Bartolini and Dixit (1991) use the option approach to evaluate illiquid bonds, which they consider an appropriate classification for sovereign debt. Claessens and Van Wijnberger (1993) apply this technique for Mexico, but instead of penalty, as in Cohen's case, they take the amount

of money generated by the sovereign economy's external sector, similar to Bartolini and Dixit. All these three applications focus on the debt service problem and renegotiation, but since the Brady plan the problem has been less one of short term finance than long term. Since debt has been replaced by bonds, the problem for sovereign debtors has been the management of reserves, and the term of the new debt issued. In this new environment, where we should add the huge capital flow to emerging countries, reserves play a more important role in external payment capacity than does trade balance.

This work extends sovereign debt evaluation models in a structured form, using an option approach similar to Das (1995) and Longstaff and Schwartz (1995) with stochastic interest rates, to cope with the new environment where loans are replaced by bonds. We assume that the sovereign bond is worth the expected value of the minimum between bond face value and the value of the country's international reserves at the bond maturity. As long as the country has enough reserves it will pay its obligation, so (similar to Claessens and Van Wijnberger) we have no room for non-excusable default. Reserves constitutes a very convenient variable, because they are observable and reflect most of the external side dynamics and the country's ability to pay.

We obtain a surprising result on the effect of US interest rate on the SOT. This model gives an indication of optimum bond maturity, size of reserves, and share of debt with fixed and floating rate, providing a framework for borrowing strategies. It also quantifies the effect of interest rate and the domestic factor volatility on SOT.

This article is divided as follows: section I presents the model; section II an empirical test; and section III the effect of the US interest rate on sovereign bond.

## I. The Model

In this model the debt current value equals the expected value of the minimum between the country's payment capacity and the final bond value. As a *proxy* for payment capacity we took the amount of reserves. A lognormal distribution is assumed for reserves, to match existing option models assumptions. We decided for the level of reserves because it is an observable variable, very sensitive to the external and internal macroeconomic scenario and at the end of the day is the money used to pay the bond. For simplicity, we also assume that all debt is represented by a bond with has only one maturity and does not pay coupon. The bond may pay a fixed, floating or a combination of both types of interest. All rates are in continuous time. The current bond value is obtained as an option where reserves is the underlying asset<sup>9</sup>, using the result of Amin and Jarrow (1992) to calculate this option in an economy with stochastic interest rate.

The behavior and evolution of reserves ( $R$ ), here defined as the risky asset, is assumed to satisfy the following stochastic equation:

$$dR(t) = \mathbf{m}R(t)dt + \mathbf{d}_1 R(t)dW_1(t) + \mathbf{d}_2 R(t)dW_2(t), \quad (1)$$

where  $W_1$  and  $W_2$  are Wiener processes that represent, respectively, shocks in the US interest rate and domestic shocks, and  $\mathbf{d}_1$  and  $\mathbf{d}_2$  are the above processes variance.  $\mathbf{d}_1$ ,  $\mathbf{d}_2$ ,  $\mathbf{m}$  and  $R(0)$  are positive constants.

The debt is denominated in US dollars, so we model the US term structure with the simplest possibility taking only one source of randomness and assumed a constant volatility process (as in Ho and Lee(1986)). The forward US interest rate in a simple one factor model can be described as:

$$df(t, T) = \mathbf{a}(t, T)dt + \mathbf{s}(t, T)dW_1(t) \quad (2)$$

where  $f$  is the forward US interest rate between time  $t$  and  $T$ , and  $W_1$  is the same as in equation (1) above.

The constant volatility assumption implies  $\mathbf{s}(t, T) = \mathbf{s}$ .

The instantaneous spot interest rate  $r(t)$  is expressed at any time  $y$  by the forward rate as  $r(y) = f(y, y)$ , therefore the riskless bond with final value \$1 and maturity  $T$  ( $\{f(t, T): T \in [t, T]\}$ ), is priced in  $t$ , as:

$$P(t, T) = e^{\left[-\int_t^T f(t, y)dy\right]} \quad (3)$$

Taking the result of Amin e Jarrow (1992) for risky assets in an economy with stochastic interest rate, we have the following equation for the bond forward price:

$$P(t, T) = \frac{P(0, T)}{P(0, t)} e^{\left\{\int_t^T \int_0^t \mathbf{s}(y, s) \mathbf{a}(y, s) dy ds - \int_0^t \left[ \int_t^T \mathbf{s}(y, s) ds \right] d\tilde{W}_1(y)\right\}} \quad (4a)$$

where

$$\mathbf{a}(t, T) = -\int_t^T \mathbf{s}(t, v) dv = -\mathbf{s}(T - t). \quad (4b)$$

The risk neutral evolution of reserves, is:

$$R(T) = R(0)B(T) e^{\left\{-\frac{1}{2}[d_1^2 + d_2^2]T + d_1\tilde{W}_1(T) + d_2\tilde{W}_2(T)\right\}} \quad (5)$$

where:  $\tilde{W}_1(1), \tilde{W}_2(T)$  are risk neutral probabilities and  $B(T)$  is the accumulated factor, corresponding to a continuously rolled over money market account, between date  $0$  e  $T$ , calculated by:

$$B(T) = \frac{1}{P(0,T)} e^{\left\{ \frac{1}{2} \int_0^T a(s,T)^2 ds - \int_0^T a(s,T) d\tilde{W}_1(s) \right\}} \quad (6a)$$

Replacing (4b) in the equation above, we have:

$$B(T) = \frac{1}{P(0,T)} e^{\left\{ \frac{\mathbf{s}^2 T^3}{6} + \int_0^T \mathbf{s}(T-t) d\tilde{W}_1(t) \right\}} \quad (6b)$$

We defined a European call option on a risk asset  $R(t)$ , with exercise price  $F$  and maturity  $T$  as  $(D(t,T))$ . Its price in a stochastic interest rate economy with constant volatility is:

$$D(0,T) = \tilde{E} \left\{ \max [ R(T) - F, 0 ] / B(T) \right\} \quad (7)$$

$$= R(0)\Phi(K) - FP(0,T)\Phi(K - \mathbf{y}) \quad (8)$$

where  $\tilde{E}$  is the expectation under risk neutrality and  $\Phi$  is the cumulated normal distribution.

$$K = \frac{\text{Ln} \left[ \frac{R(0)}{FP(0,T)} \right] + \frac{1}{2} \mathbf{y}^2}{\mathbf{y}}$$

$$\mathbf{y}^2 = (\mathbf{d}_1^2 + \mathbf{d}_2^2)T + \mathbf{d}_1 \mathbf{s} T^2 + \frac{\mathbf{s}^2 T^3}{3}$$

For practical purposes we can distinguish three different cases when we apply this model. The first, when the debt pays only fixed interest, the second when the interest paid is floating according to some market interest rate and the last one a combination of fixed and floating.

**Case 1.** We take the final bond value with a fixed payment  $F$  and maturity date  $T$  ( $D(0,T)$ ) as an option where the debtor country pays either the minimum of the amount of reserves or the bond face value. In other words, if the country has enough reserves it pays the full amount of the debt, otherwise it pays whatever reserves it has. The bond current value can be easily described in option style.

$$D(0,T) = \tilde{E}\{ \text{Min}[ R(T), F ] / B(T) \}. \quad (9)$$

$$\begin{aligned} D(0,T) &= \tilde{E}\{ [ F - \text{Max}(0, F - R(T)) ] / B(T) \} \\ &= F P(0,T) - \tilde{E}\{ \text{Max}(0, F - R(T)) / B(T) \}. \end{aligned} \quad (10a)$$

The risky debt equals a risk free investment with final value  $F$  minus a put option, so the bondholder is long a risk free bond and short a put option. If nothing bad happens (this scenario is the one where reserves is higher than bond final value -  $F$ ), the investor *receives the principal back, otherwise loses part or all money invested*. The risk premium equals the current value of the put option that the bondholder is short. Based on Put-Call-Parity (which is the mathematical relationship between a call and put option) the above equation can be written as:

$$= R(0) - \tilde{E}\{ \text{Max}(0, R(T) - F) / B(T) \} \quad (10b)$$

Using equations (7) and (8), the current bond value is calculated as:

$$D(0,T) = R(0) - R(0)\Phi(K) + FP(0,T)\Phi(K - y) \quad , \quad (11)$$

The rate paid by this bond is easily calculated as  $i_d = -\left(\frac{1}{T}\right) \text{Ln}\left(\frac{1}{D(0,T)}\right)$ , and we defined the risk premium as the difference between the bond internal rate of return and the US interest rate ( $r_d = i_d - i_{US}$ ) with the same maturity.

The risk premium increases when domestic volatility, defined by  $d_2$ , and the US interest rate volatility increase. The correlation between reserves and the US interest rate, defined by  $d_1$ , has a negative impact on risk premium. The higher the level of reserves, obviously, the lower the risk premium. A not so intuitive result is obtained with the level of the US interest rate, the higher the interest rate level, the lower the risk premium. For some<sup>10</sup>, the effect is the inverse, but for others (Das (1995) and Longstaff and Schwartz (1995)) this a regular result in corporate bond analysis. The reason for this is that an increase in  $i_{US}$  implies a higher drift of the risk neutral process for  $R(t)$ , and  $R(t)$  is expected to move away from  $F$  at a faster rate, which increases the option value and decreases the risk premium. The magnitude of the risk premium reduction depends on the value of  $d_1$ .

**Case 2.** The bond pays floating interest rate, its current value is:

$$\begin{aligned} D_f(0,T) &= \tilde{E}\left\{ \text{Min}\left[ R(T), F_0 B(T) \right] / B(T) \right\} \\ &= \tilde{E}\left[ \frac{R(T)}{B(T)} - \text{Max}\left( 0, \frac{R(T)}{B(T)} - F_0 \right) \right]. \end{aligned} \quad (12)$$

where  $F_0$  is the initial bond face value.

Again, taking the model of Amin and Jarrow we obtain:

$$D_f(0,T) = R(0) - \tilde{E}\left[ \text{Max}\left( 0, \frac{R(T)}{B(T)} - F_0 \right) \right]. \quad (13)$$

The second term of the above equation is an option, and its solution is in appendix

A. The current bond value is:

$$D_f(0, T) = R(0) - R(0)\Phi(K1) + F_0 \Phi(K1 - y1), \quad (14)$$

$$K1 = \frac{\text{Ln}\left[\frac{R(0)}{F_0}\right] + \frac{1}{2}y1^2}{y1},$$

$$y1^2 = (d_1^2 + d_2^2)T$$

The risk premium is obtained directly from the difference between  $D_f(0, T)$  and  $F$ , which in continuous time and annual rate is:

$$r_d = -\left(\frac{1}{T}\right) \text{Ln}\left(\frac{D_f(0, T)}{F_0}\right). \quad (15)$$

**Case 3.** In the case of a mixed debt with fixed and floating rate payments, we assume a total debt of  $F$ , split into  $F1$  of floating rate and  $F2$  of fixed rate.

$$F1 = xF \quad \therefore \quad F2 = \frac{(1-x)F}{P(0, T)} \quad (16)$$

The task is to obtain the current value of the total debt  $F$ , which we call  $D_c(0, T)$ .

Therefore, we solve the following option problem:

$$D_c(0, T) = E\left\{ \text{Min}\left[ R(T), xFB(T) + \frac{(1-x)F}{P(0, T)} \right] / B(T) \right\} \quad (17)$$

Due to the difficulty in calculating the sum of two lognormal variables, we opted to calculate this option value via Monte Carlo, pursuant to the equations in appendix B. The risk premium is calculated as:

$$r_d = -\left(\frac{1}{T}\right) \text{Ln}\left(\frac{D_c(0,T)}{F1 + F2}\right) \quad (18)$$

Among the very interesting results that can be obtained from (17), we can get an optimum combination of reserves, floating versus fixed interest rate and term of the debt. If the objective function is to minimize the risk premium, we just need to find the  $R(0)$ ,  $x$  and  $T$  in (18) that gives this minimum.

## II. Model Fitting

The first step is to estimate the parameters of the stochastic process of reserves and US interest rate, from equations (1) and (2), respectively. We estimate these continuous time model parameters using Maximum Likelihood-ML. Using Ito's lemma, equation 1 can be represented as:

$$d\text{Ln}(R) = \left( \mathbf{m} - \frac{\mathbf{d}_1^2 + \mathbf{d}_2^2}{2} \right) dt + \mathbf{d}_1 dW_1 + \mathbf{d}_2 dW_2 \quad (19)$$

For equation 2:

$$df = \mathbf{a} dt + \mathbf{s} dW_1 \quad (20)$$

From (19) and (20) we have two normal variables:

$$\text{Ln}\left(\frac{R_{t+\Delta}}{R_t}\right) \approx N(\mathbf{g}\Delta, \mathbf{s}_R^2 \Delta) \quad \therefore \quad f_{t+\Delta} - f_t \approx N(\Delta t, \mathbf{s}^2 \Delta) \quad (21)$$

where:

$$\mathbf{g} = \mathbf{m} - \frac{\mathbf{d}_1^2 + \mathbf{d}_2^2}{2} \quad \therefore \quad \mathbf{s}_R^2 = \mathbf{d}_1^2 + \mathbf{d}_2^2$$

From equations (21) we have a normal bivariate and want to estimate the parameters vector  $\mathbf{q} = \{\mathbf{g}, \mathbf{a}, \mathbf{s}, \mathbf{s}_R, \mathbf{r}\}$ , from the joint density function. Applying ML we have the estimators below:

$$\mathbf{g} = \frac{\left( \sum_{j=1}^T \text{Ln} \left( \frac{R_{j+1}}{R_j} \right) \right)}{T}$$

$$\mathbf{s}_R^2 = \frac{\sum_{j=1}^T \text{Ln} \left( \frac{R_{j+1}}{R_j} - \mathbf{g} \right)^2}{T}$$

$$\mathbf{a} = \frac{\sum_{j=1}^T (f_{j+1} - f_j)}{T}$$

$$\mathbf{s}^2 = \frac{\sum_{j=1}^T (f_{j+1} - f_j - \mathbf{a})^2}{T}$$

$$\mathbf{r} = \frac{\sum_{j=1}^T \left[ \text{Ln} \left( \frac{R_{j+1}}{R_j} - \mathbf{g} \right) (f_{j+1} - f_j - \mathbf{a}) \right]}{T \mathbf{s} \mathbf{s}_R}$$

where T is the number of observations.

Based on these statistics we can easily find the original parameters in equation (1), using:

$$m - \frac{d_1^2 + d_2^2}{2} = g$$

$$d_1^2 + d_2^2 = s_R^2$$

$$\text{Covariance}(d\text{Ln}(R), df) = r s_R s$$

Assuming a zero correlation between  $dW_1$  and  $dW_2$ , it is easy to show :

$$\text{Covariance}(d\text{Ln}(R), df) = d_1 s$$

The solution of this system is:

$$m = \frac{s_R^2}{2} + g$$

$$d_1 = s_R r$$

$$d_2 = s_R \sqrt{(1-r)}$$

We calculated these parameters for Brazil, using monthly observations with an arbitrary sample of seven years. The US forward rate was approximated by the US rate embedded in 10 and 30 years zero coupon bonds. For a sample ending in December and June of each year from 1995 to December 1997, the calculated parameters are shown in table I.

Taking the brazilian debt size, amount of reserves, percentage of floating to fixed interest, and debt duration, we can apply the result from (18) and calculate the theoretical brazilian risk premium. For the US interest rate we simply took the rate of 10 years zero coupon<sup>11</sup>. These numbers are in table II.

We can compare the theoretical risk premium from (18) to the market risk premium obtained from a traded bond. We took the C-bond because it is the most liquid and has

no interest guarantee (which makes calculation easier), the calculation procedures for its implied risk premium are in appendix C.

The results in table III are encouraging except for 1996, when market risk premium goes down dramatically, while the model still shows a high risk premium. Two types of reasons can explain this difference. The first is that this model is not adequate and the second is the use improper parameters. The first, will depend on the hypotheses of this option model, we may be using an inadequate process for interest rates and the reserves, and/or arbitrage with the underlined asset (reserves) may not be possible, but if one can get a long or short position on reserves and the processes follow (1) and (2), it is easy to show that (17) holds. The parameters calculation is a difficult task, especially the volatility. The market evaluation of volatility between evaluation date and maturity can be very different from the historical one. The size of reserves can also be misleading because of a market perception that in case of emergency the IMF can provide some extra money, therefore the reserves are potentially higher<sup>12</sup>. Finally, another reason is that we presented a simplification, with no coupon and only one maturity date, which can affect the result.

Based on data from December 1996, it is possible to determine the best combination of the percentage of fixed/floating and debt term that Brazil should have for a minimum risk premium. In figure 1 we show a hyperplane with this combination.

It is easy to show (and can be verified from the figure 1), that the lowest risk premium occurs at the longest term and 100% floating interest. The reason is that in a risk neutral evaluation, the probability of default goes down with time. Another interesting result is the risk premium convexity embedded in the fixed interest rate bond,

which points to an optimum maturity in the medium term. The inclusion of the stochastic interest rate economy increase the risk premium via a positive  $\mathbf{s}$ .

Some conditional economic recommendation can be obtained from this model. For example the Brazilian government may choose the best combination of term and percentage of floating debt and may also act by diminishing the volatility provided by  $\mathbf{d}_2$  and increasing reserves (up to the point where the risk premium decrease equals the cost to raise reserves).

### **III. The Effect of the US Interest Rate on Risk Premium**

A quick look at the risk premium variation versus the US interest rate variation shows a positive correlation. In fact the correlation between risk premium and the US interest variation is 49% for a sample ranging from March-1995 to February 1997. In figure 2, one notices a consistent and strong decline in Brazilian risk premium.

Although a first look shows positive correlation, one should distinguish between US interest rate shocks and a not surprising increase already embedded in term structure. A positive shock affects the risk premium positively, but a higher interest rate level decreases the risk premium due to the risk neutral valuation approach of this model. This sample correlation can also be misleading, because doesn't take in account other country specific factors.

One interesting application is the protection of a sovereign bond portfolio against the US rate variation. The portfolio can be split into two factors: risk premium and the US interest rate. The hedge of the latter factor is straightforward with regular

futures/options, but these instruments can also be useful for risk premium hedge. Hedge ratios can be obtained from sensitivity analysis between risk (equation 17) and the US interest rate.

In Telljohann (1994), published by the Chicago Board of Trade-CBOT for practitioners in the Brady Bond market, there are three different strategies to protect *Brady Bonds* against the US interest rate variation. The technique used was to eliminate the effect of US rate by adding an interest rate future contract traded in CBOT to offset the effect of the US interest rate variation on the Brady Bond. This effect is calculated based on a historical data series and three different hypotheses about the effect of the US rate on the country's risk premium. Here we provide a model to explain this relationship analytically.

#### **IV. Summary**

The result obtained above shows that Brazilian bonds have been overvalued if one takes historical volatilities and the current amount of reserves. It is well known in option model applications that market volatilities can be highly different from historical ones, because the important thing is future volatility rather than what happened in the past, although the model assumes constant volatility.

This sheds light on the long run perspectives of sovereign bonds return, as Krugman (1995) suggests that we may have some market overvaluation.

This paper can be extended in many different ways: a different specification for US interest rate processes (equation 3), reserves behavior, different degrees of guarantee and seniority for the bonds, and inclusion of coupon payments. It can also be useful for

several applications: empirical testing of hedge strategies, modeling other bonds and countries, evaluation of new countries entering the market, impact of IMF bailout, debt renegotiation and modeling sovereign bonds derivatives.

### Appendix A - Option Value Calculation

Based on Amin and Jarrow we have:

$$\begin{aligned} & \tilde{E} \left[ \text{Max} \left( 0, \frac{R(T)}{B(T)} - F_0 \right) \right] \\ & R(T) = R(0) B(T) e^{\left\{ -\frac{1}{2} [d_1^2 + d_2^2] T + d_1 \tilde{W}_1(T) + d_2 \tilde{W}_2(T) \right\}} \end{aligned} \quad (\text{A.1})$$

where  $d_1 \tilde{W}_1(T) + d_2 \tilde{W}_2(T)$  and  $\text{normal}(0, [d_1^2 + d_2^2] T)$

Taking  $x = -\frac{1}{2} [d_1^2 + d_2^2] T + d_1 \tilde{W}_1(T) + d_2 \tilde{W}_2(T)$ , we look for:

$$\tilde{E} [R(0) e^{\tilde{x}} - F_0]^+ = R(0) \left[ \int_{F_0/R(0)}^{\infty} e^x f(e^x) d(e^x) - \frac{F_0}{R(0)} \int_{F_0/R(0)}^{\infty} f(e^x) d(e^x) \right]$$

The function  $f$  is a lognormal density and the solution of the above equation, as in Ingersoll (1987, p. 14) is:

$$R(0) \left\{ e^{\left( m_x + \frac{s_x^2}{2} \right)} \Phi \left( \frac{m_x - \text{Ln} \left( \frac{F_0}{R(0)} \right)}{s_x} + s_x \right) - \frac{F_0}{R(0)} \Phi \left( \frac{m_x - \text{Ln} \left( \frac{F_0}{R(0)} \right)}{s_x} \right) \right\}$$

where  $m_x$  and  $s_x$  are the mean and standard deviation of  $x$ .

With proper change of variables for the mean and variance of  $x$  we have formula 13.

## Appendix B - Option Valuation via Monte Carlo

From equation 17, we have:

$$D_c(0, T) = \tilde{E} \left\{ \text{Min} \left[ \frac{R(T)}{B(T)}, xF + \frac{(1-x)F}{P(0, T) B(T)} \right] \right\} \quad (\text{B.1})$$

and from (5) and (6b):

$$\frac{R(T)}{B(T)} = R(0) e^{\left\{ -\frac{1}{2} [d_1^2 + d_2^2] T + d_1 \tilde{W}_1(T) + d_2 \tilde{W}_2(T) \right\}} \quad (\text{B.2})$$

$$B(T) = \frac{1}{P(0, T)} e^{\left\{ \frac{\mathbf{s}^2 T^3}{6} + \int_0^T \mathbf{s}(T-t) d\tilde{W}_1(t) \right\}} \quad (\text{B.3})$$

The variable  $\int_0^T \mathbf{s}(T-t) d\tilde{W}_1(t)$  is  $N\left(0, \int_0^T \mathbf{s}^2 (T-t)^2 d\tilde{W}_1(t)\right) \approx N\left(0, \frac{\mathbf{s}^2 T^3}{3}\right)$ .

Taking  $\mathbf{e}_i$  as a stochastic number extracted from a random sample with mean 0 and variance 1, we generated two sources of random variation:

$$\tilde{W}_2(T) = \mathbf{e}_i \sqrt{T} \quad (\text{B.4})$$

In order to estimate the second term inside the exponential on equation B.3, we simulated this process at the same time as in B.2.

$$\tilde{W}_1(T) = \mathbf{e}_i \sqrt{T} \quad (\text{B.5})$$

$$\int_0^T \mathbf{s}(T-t) d\tilde{W}_1(t) = \mathbf{s} T \mathbf{e}_i \sqrt{\frac{T}{3}} \quad (\text{B.6})$$

With these equations we generated a sample for B.2 and B.3, and consequently B.1.

The final price in equation B.1 was calculated as the presented value (risk neutral

valuation) of the expected value of the option value at maturity. For our examples we generated 1000 random numbers. For a discussion on Monte Carlo application to option valuation, see Boyle (1977) or Hull (1992).

### **Appendix C - C-bond Risk Premium Calculation**

The C-bond (Capitalization Bond) was issued in 15/4/94 and matures 15/4/2014. It has no guarantee of any kind. The cash flow is presented in the table below.

Interest is paid semi-annually on April and October 15 of each year. For the first two years, interest rate is fixed at 4% pa, the next is 4.5% pa, other two years 5% pa and the rest of paper's life the interest rate is fixed at 8% pa. Amortization starts at 04/15/2004 at 5.86% semi-annual.

The implied continuous rate of return on this bond is simply the rate that equals the present value of the bond to its market price, as stated in the equation below:

$$P = \sum_{j=1}^N C_j e^{-rt_j} \quad (\text{C.1})$$

where  $C_j$  is each total payment at date  $j$ .

$t_j$  is time length from evaluation date to payment date  $j$  in annual terms.

$P$  is the dirty price (as explained below).

The prices shown in the financial market do not include accrued interest (*clean price- $P_c$* ), but at moment of payment the accrued interest (also known as *dirty price- $P_D$* ) should be added.

The risk premium embedded in this bond is usually quantified as the difference between the rate  $r$  and the US rate with the maturity (or duration). We called this risk premium  $r_B$ .

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**Table I****Estimates of Interest Rate and Reserves Process Parameters**

Each estimation was done with seven years of monthly data ending June or December of each year presented below. The parameters are estimated by the Maximum Likelihood with t-statistics in parentheses.

	1995		1996	
	June	December	June	December
$\mu$	0.02120	0.02345	0.02597	0.02494
$\alpha$	(0.00033)	(0.00038)	(0.00023)	(0.00018)
$\sigma$	0.00232	0.00231	0.00237	0.00233
$\delta_1$	0.01514	0.01625	0.01253	0.01194
$\delta_2$	0.07954	0.08169	0.07941	0.07844

**Table II****Data for option calculation.**

Source:  $F$ ,  $R(0)$ ,  $x$  and  $T$  come from the Brazilian Central Bank Bulletin and the interest rate for US treasury (zero coupon) with 10 years from the *Federal Reserve Bulletin*. All values are in US\$ billions and  $T$  in years. The US interest rate is in yield form. The total debt is calculated based on current face value.

	<b>1995</b>		<b>1996</b>	
	<b>June</b>	<b>December</b>	<b>June</b>	<b>December</b>
<b>F</b>	124.9	129.3	135.2	144.1
<b>R(0)</b>	33.5	51.8	60.0	60.1
<b>x</b>	46.7%	46.5%	45.9%	44.1%
<b>T</b>	8.3	8.1	7.6	7.6
<b>i<sub>US_10</sub></b>	6.17%	5.71%	6.91%	6.30%

**Table III**

**Theoretical versus Implied Risk Premium in C-Bond Prices**

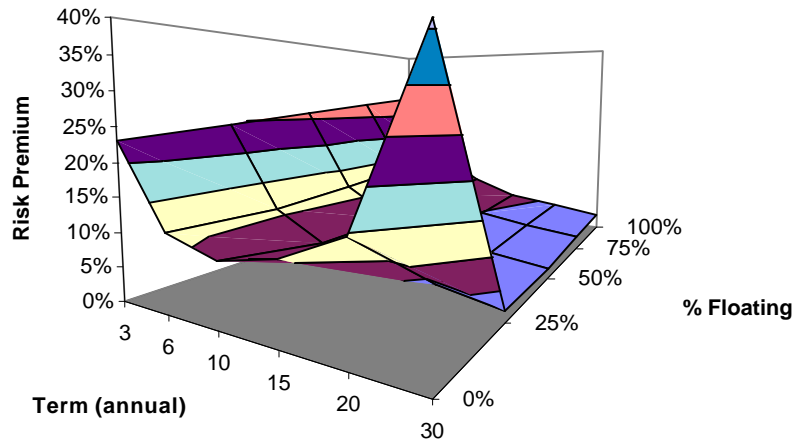
The lines show the result of our model, using the data presented in table I and II. The second line show the C-bond implied risk premium calculated accordingly appendix C. The last line show the C-bond prices used for the implied risk premium calculation. They were collected at the end of the month.

	<b>1995</b>		<b>1996</b>	
	<b>June</b>	<b>December</b>	<b>June</b>	<b>December</b>
<b>Theoretical risk premium</b>	14.49%	11.23%	10.28%	11.00%
<b>C-bond implied risk premium</b>	10.84%	9.58%	7.68%	5.75%
<b>C-bond Clean Price</b>	49.06	57.19	61.94	73.75

**Figure 1**

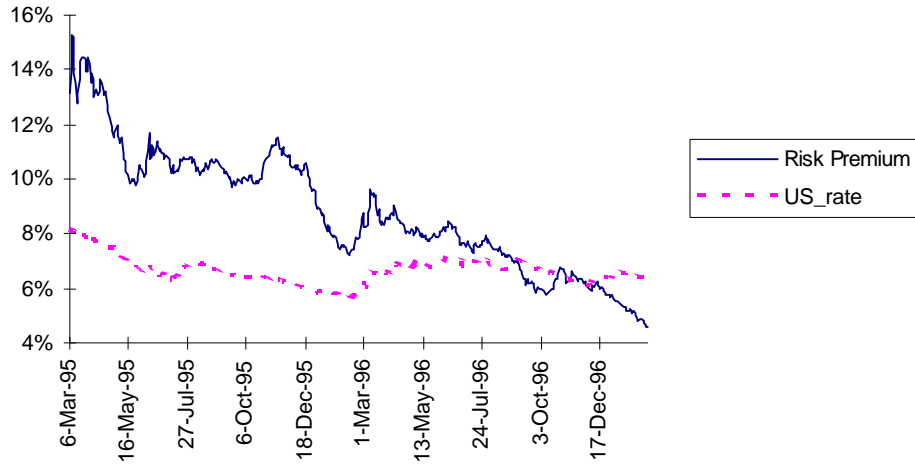
**Risk Premium versus Term and Percentage of Floating Interest Bonds.**

This a mapping of the theoretical risk premium for different debt term and percentage of floating (in the total amount of debt). This calculation is based on estimated parameters for Brazil as of December of 1996. These parameters are in table I and II. This table shows a point of minimum risk premium at a term of 30 years (maximum considered) and 25% of floating rate debt.



**Figure 2**

**Implied Risk Premium in C-Bond versus the US interest Rate**



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<sup>1</sup> The name Brady is due to the US Treasury Secretary Nicholas Brady, who first proposed this type of plan in 1989.

<sup>2</sup> As Kaletsky (1985) shows, the default triggers many mechanisms that make rescheduling costly.

<sup>3</sup> During the recent Mexican crisis (in 1995), even the US government participated in a loan to Mexico.

<sup>4</sup> Penalties can be in the form of credit constraint, interference in international trade and capture of external assets.

<sup>5</sup> The spread over treasury, as the name indicates, is in general, defined as the difference between the yield on sovereign bond and the US Treasury bond with similar maturity. It represents the risk premium received by the creditor to take the issuer credit risk. It is also known as credit spread.

<sup>6</sup> See Bartholomew (1991), Cumby and Evans (1993), Hernandez-Trillo (1995) and Claessens and Pennacchi (1996).

<sup>7</sup> In fact the market trades sovereign and corporate bonds with similar credit rating criteria, therefore it seems reasonable to use the same technology to evaluate them.

<sup>8</sup> More precisely, he assumes that the banks can oblige the country to pay out a fraction of the tradable sector.

<sup>9</sup> Although the underlying asset is not a traded asset, it is likely spanned by traded instruments. For example, exports can be partially spanned by futures and forward contracts on commodities, foreign direct investment by stock market, etc. It does not

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seem an easy task, but possibly some arbitrage is achievable. Assuming it is possible, then we have complete markets and therefore we can apply risk neutral valuation techniques.

<sup>10</sup> Calvo *et all* (1993) stress the importance of the US interest rate decline to explain a huge flow of capital to emerging markets, especially Latin countries.

<sup>11</sup> The floating rate Brazilian bonds pay LIBOR, but for simplicity we used US treasury implied yield in continuous time format.

<sup>12</sup> For example, in the case above, if Brazil is able to borrow an extra US\$60 billion with the same conditions as the current debt, its theoretical risk premium should go down to 5.53%.